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Steps in the Philosophy of Quantum Theory

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Abstract: *1. Interpretation.* The Copenhagen Interpretation (CI) is a minimal semantics to quantum theory, expressing what we know at least. It can be extended into a universal Quantum Theory, applied to the observer as well as to the observed object. *2. A Universal Theory as a Philosophical Problem.* A circular epistemology is proposed, consisting of non-hierarchical realism, empirism, apriorism and evolutionism, combined in a description of time: past as discrete facts, future as continuous possibilities. *3. Quantum Logic and the Reconstruction of Quantum Theory.* Non-distributive logic and Bell's theorem are discussed following Doebner and Lücke. Reconstruction is briefly described. *4. Further Philosophical Questions.* Mind-body problem and holism are briefly discussed.

1 Interpretation

1.1 Steps in Quantum Theory

"Can you tell me what you mean by what you say?" This is the philosophical question, asked by Socrates to the Athenians and by his re-incarnation Niels Bohr to the physicists. Philosophy is historically a posteriori. Philosophy of science asks questions about science already invented. Steps in science seem to come first.

Classical physics was a *continuum* dynamics. In point mechanics, observables like energy, momentum, position, angular momentum admitted a continuum of values. Field mechanics even had an infinite number of degrees of freedom.

Statistical mechanics, as Boltzmann realized, needed *rigid atoms* in order to permit thermodynamical equilibrium.

Planck, for the same reason, applied to the Maxwell field, needed the *quantum of action*, i.e. *discrete energy levels* for the oscillator.

Einstein probably clearly saw the *impossibility* of *any continuum dynamics* for fields, and introduced the *photon*.

Bohr, receiving from Rutherford the first credible model of an *atom*, saw that it was possible only with *discrete energy levels*, too.

Heisenberg introduced the *algebra* of observables, Schrödinger the *wave function*, Born interpreted the wave as expressing *probabilities*, and von Neumann united the mathematical concepts in a *Hilbert space*. Within a separable Hilbert space, observables possessing eigenfunctions have only *discrete* eigenvalue spectra.

It is the von Neumann codification of quantum theory to which the later interpretation debate referred.

1.2 The Interpretation Debate

Let us first describe the more than sixty years of interpretation debate by a parable.

Heisenberg [13] said in connection with quantum theory: "In modern physics, nature now has reminded us clearly that we can never hope to open up the complete field of possible knowledge by starting from a fixed basis of operations. Rather for every essentially new insight we will again come into the situation of Columbus who dared to leave all so far known land in the nearly manic hope (in der fast wahnsinnigen Hoffnung) to find land again beyond the seas." Later, in 1948, he described the process of theoretical physics as an open sequence of "closed theories" ("abgeschlossene Theorien") in which the later one always explains and thus limits the success of its predecessors. Thomas Kuhn [15] described the process as a sequence of paradigms which are used but not fundamentally understood, and of scientific revolutions, arising of taking seriously the interpretational contradictions within the earlier paradigm. Which step beyond quantum theory might be foreshadowed by the interpretation debate?

Heisenberg's parable of Columbus is methodologically not precise. Columbus knew, as Greek astronomy had already known, that the earth is a sphere. His mania was not in the absolutely correct idea that going westward one would come to India. His mania was in daring to do the traveling himself with the available nautical means, and his reward was to discover an unexpected continent for which there had been space enough between the oceans. The inventors of quantum theory were in a less satisfactory position. They discovered and conquered an unexpected continent, but they did not know whether the field of physical knowledge is a sphere, and if so, where the new continent is located on it, and whether there are more continents to be discovered.

1.3 The Copenhagen Interpretation

The Copenhagen interpretation (CI) is an attempt to describe consistently the structure of the new continent without making hypotheses about its location with respect to unknown continents or to the complete field of possible knowledge, not to speak of realities which remain unknown to human beings in principle. CI is epistemological, not ontological. It can be described as aiming at a *minimal semantics* for the quantumtheoretical formalism.

In CI, Quantum theory is a theory on available or possible *human knowledge.* In Hilbert space there are vectors and operators. The operators are possible observables, described as Hamiltonians of measurement interaction (see [21, pp. 531-534] and [9,10]). The vectors (Ψ-functions) define probabilities of finding eigenvalues of the observables.

This solves, by the way, the problem of continuum dynamics: The measurable quantities have *discrete* values, the *continuum* defines probabilities, thus not leading to an ultraviolet catastrophe. A remark is relevant on operators with a continuous spectrum like position, momentum, field properties. The claim of eigenstates to such a spectrum cannot be fulfilled in a separable Hilbert space. We propose to consider the definitions of such continuous observables as only approximately valid. This will be discussed in the chapter on reconstruction.

Probabilities which are determined by law of nature are essentially *conditional probabilities.* In the *quantum theory of measurement* this is described by distinguishing between *preparation*, as defining the condition, and *observation*, as defining the outcome. The result of an observation which had a probability different from one will change the condition and hence the probabilities. This is called the reduction (or, more dramatically, the *collapse*) of Ψ. Thus, in CI, Ψ is essentially an *expression of knowledge.*

This makes inevitable the question of the relationship between *observer* and *object*, between knower and known. Measurement is done by physical interaction between observer and instrument. Are we to describe the observer, too, by quantum theory? If yes, how? If no, why not?

Bohr answered No. He spoke of the "detached observer". This can be methodologically justified by considering CI as minimal semantics. We can express our knowledge of the objects without having a theory on how this knowledge as a mental act is to be objectively described. Bohr himself had an additional ontological reason. At least in his younger years he did not believe that living organisms can be fully described by physics; and he never believed that human consciousness can be so described. We shall deviate from these views of Bohr's; but we continue to accept the CI minimal semantics as a meaningful step of interpretation.

Then the question remains, how far the minimal semantics will force us or at least admit to describe the measurement process by quantum theory (compare [22,23]). Bohr insisted that the instrument must be classically described: A measurement means description in intuitive space-time, and it presupposes strict causality between the observed effects and the state of the object which we want to know. (This is a good Kantian argument.) Even in Bohr's view this did not necessarily mean that quantum theory should not be valid in the instrument; it suffices that the conclusions to be drawn from the observation are classical in the necessary approximation.

CI as a minimal semantics is also not troubled by the famous question: "When is Ψ reduced in the act of measurement?" Since Ψ, according to CI, expresses human knowledge, no harm is done by saying: "It is reduced when the observer becomes aware of the result." But Bohr's postulate of a classical description justifies equally the "Golden Copenhagen Rule": "No harm is done in assuming that Ψ is reduced when an irreversible process has taken place in the measuring instrument." This eliminates the impression of "subjectivity", since an irreversible fact is in principle accessible to every observer.

1.4 Universal Quantum Theory

Today it seems to be a possible second step after CI, to fully accept the unchanged von Neumann formalism, but to apply it to all real events without restriction, as a universal theory. Hypothetically to retain the formalism seems justified by its success through 60 years, and by the failure of the only competitor which could be experimentally tested: the theory of local hidden variables. Hypothetically to extend it to universal validity seems justified by the success of physicalism in biology and by progress in cosmology.

The question is whether universal quantum theory admits an interpretation without paradoxes. This question leads into two problem fields:

1. Can and must CI be replaced by a different interpretation?
2. Does "universal" mean to apply the theory also to the mind of the observer?

Problem 1. Several proposals have been made which maintain to retain the mathematical structure and the experimental results of quantum theory but not the Copenhagen Interpretation. We have discussed the proposals made by Kochen [14], by Deutsch [4] who follows Everett [7], and by Cramer [3]; this discussion was given in our papers [9–11]. We shall not repeat it here, but mention our conclusion.

We maintain:

A. If two theories are mathematically isomorphic and predict identical experimental results, then there must be a possible dictionary translating their verbal expressions into each other. It must then be possible to consider them as different formulations of one theory, looking at it from different directions. The question then is how to interpret this theory in the traditional language. Since CI is the minimal semantics of quantum theory, the new proposals must be expressible in the CI language as far as CI can be applied. Hence they will not replace, only contribute to CI if CI can be extended to universal validity. This will lead to Probl. 2.

B. Attempts have been made to change not only the interpretation but also the formalism of quantum theory, mainly in order to avoid apparent paradoxes in the theory of measurement. The main problem arose from considering Ψ not epistemologically but as an "objective reality"; this view then needed an explanation of the "collapse of Ψ" by observation. One proposal – local hidden variables – was experimentally excluded; others still await a test or will not admit an experimental distinction from quantum theory in a foreseeable future (e.g. Ghirardi et al. [8]). In the present paper we shall not consider these proposals since we feel that existing quantum theory permits a consistent interpretation.

Problem 2. Here the problem is not whether quantum theory can be applied to the human brain; if we presuppose physicalism as correct in biology as most biologists do today, then the application to the human brain is a consequence. The task is to eliminate the so-called mind-body problem. This is automatically achieved if we "reconstruct" quantum theory as a general theory for predictions on *any* *empirically decidable alternatives*: "abstract quantum theory", cf. [21, Chapt. 8],[6]. The only precondition (and hence, limitation) of this view lies in the assumption that mental states can be objectively ("decidably") observed by introspection and communication. We shall return to the philosophical problems of this view in Sect. 4.

2 A Universal Theory as a Philosophical Problem

2.1 The Problem

"Can you tell what you mean by trusting in a universal theory?" Columbus trusted in the earth being a sphere, with greet success. The postulates of abstract quantum theory can be formulated on a half page of print, at least for a mathematically educated reader; the physicists' community trusts today in approximately a billion single empirical facts agreeing with quantum theory and so far in none which would convincingly contradict the theory. How can such a universal theory be possible? What is the "sphere of human knowledge" which admits a universal theory?

Plato, Hume, Kant, and Popper agree that the strict validity of a universal proposition cannot be empirically proved; as Hume pointed out, because at least the future cases of empirical application of the proposition cannot be known at present. This reminds us of the basic role of *time* in empirical science; experience can be defined as learning from the past for the future. Popper's statement that a universal proposition can at least be empirically falsified by one counter-example is correct only as far we can trust the propositions in which we interpret the counter-example.

The problem is hard, and we do not propose a final solution, but steps which might be useful (cf. [21, pp. 622–627]).

2.2 Pragmatism and Hierarchism

The average attitude of scientists in view of our problem is pragmatic. "We are successful; let us continue." In Kuhn's language this is the mentality of "normal science": puzzle-solving under a successful paradigm. It is essential for this attitude, not to ask why the paradigm is so successful. The opposite is true for scientific revolutions. The paradigm of Newton's mechanics was overcome and thereby, justified within its limits, by Mach's critical analysis of its concepts and Einstein's positive answer to Mach's questions.

The opposite attitude to "normal" pragmatism may be called hierarchism. If we formulate the meaning of a paradigm in propositions, these may be used as fixed axioms, from which a scientific discipline might be logically deduced; Newton's mechanics is a good example. The origin of this kind of science is the great Greek discovery of deductive mathematics. But how to find adequate axioms for physics?

2.3 Realism and Empiricism

Physics rests on experience and speaks about reality. Thus the philosophy of physics was tempted to formulate basic axioms either on reality (ontology) or on experience (epistemology). Both ways of theory-building were intermediately successful, none of them, however, had conclusive success.

Classical physics gave us a picture of reality: matter and fields, deterministically interacting in space and time. Many physicists, down to our days, have a nostalgic longing for this picture of reality. CI, the minimal semantics of quantum theory, makes use of this picture only for describing our sensual experience, not for describing basic physical reality. Can we find a classical or semi-classical picture of reality behind CI?

Empiricism or "positivism" denies the legitimacy of this wish; it even questions the positive meaning of concepts like "reality". The Vienna school tried to make axiomatic use of the data of sense-perception. But Popper rightly pointed out, that sensual experience justifies no universal proposition. He proposed a progress of hypothetical pictures of reality which are used as long as they are not falsified by empirical counter-examples. Yet he could not explain why any "picture of reality" should have such horrend success as classical mechanics or even more quantum mechanics. Seeing his lack of explanatory power he called his view a "robust realism". It is a belief, nothing more.

2.4 Kant's Apriorism

Kant offers an answer to the question: We possess cognition a priori, which applies to the experience but does not depend on experience. But how might that be possible?

Kant's first answer: Cognition a priori is a fact in mathematics. I need no special experience for understanding that $2 \times 2 = 4$, that $17 \times 19 = 323$. I have certainly often empirically tested $2 \times 2 = 4$, but never $17 \times 19 = 323$, but I am as sure of one of the equations as of the other. We understand mathematical truths by constructing them ourselves.

Kant maintains that physics can be equally constructed a priori. The construction is done in our originary "forms of intuition", space and time, by means of our originary conceptual categories like substance and causality. Without categories no universal propositions, without universal propositions (like, e.g., the law of causality) and forms of intuition no conceptually expressible experience, hence no physics. The principles of physics apply always *in* experience because they are preconditions *of* experience. There is no contradiction between the universal laws and the behaviour of the reality as described by physics, because this reality is precisely what *we* can know. It is our construct. One might ponder whether perhaps no conceptual experience might be possible at all. Then there would be no conceptually thinking human beings. But *if* experience is possible, then it has to agree to the laws a priori.

Modern physics has made it practically impossible to be a strict Kantian. Relativity and quantum theory deny precisely those universal laws which he considered as a priori true. But it is neither our tendency to accept hierarchic theories as definite nor to deny their value as steps in a proceeding way of understanding. Realism offered the fruitful model of classical physics, positivism offered the fruitful criticism of this model, apriorism offers a hope of understanding why universal laws should hold in experience.

2.5 Evolutionism

How can we possess cognition a priori? Konrad Lorenz said: Because our ancestors have acquired it in the process of evolution as an adaptation to reality. We possess an inborn intuition of three-dimensional space because without this our ancestors, the monkeys and apes, would have fallen from the trees when jumping. Our inborn forms of cognition are adapted to reality because they are a gift of an evolution in the real world. This is, in the present authors' view, a very profound step forward in the philosophy of science. But it is certainly not, as Popper thinks, a vindication of the realism of classical physics, beyond the trivial fact that classical physics is well adapted to the macroscopic bodies with a temperature far from absolute zero, from whose perception its special structure was derived in the history of science. The jumping apes needed no more then being oriented in the same macroscopic world. Beyond this, some of our classical theories may even be products not of evolutionary inborn ideas but of the history of a special – here the occidental – civilization. Thus it is not at all clear that our spatial intuition is strictly Euclidean. In true fact it seems to be unprecise, but adaptable to that wonderful Greek invention, the Euclidean geometry.

2.6 Circular Epistemology and Quantum theory

We would conclude that neither the merely pragmatic attitude nor any strict hierarchism is adequate. There is a circle of mutual explanation through which we must go repeatedly. Nature is older than the human species; the human species is older than natural science. Our concepts are inherited from evolution and from cultural history; our description of evolution and of cultural history is done in our available concepts. Let us keep this in mind when now we return to quantum theory.

The basic idea in using this circular epistemology in interpreting quantum theory is as follows.

We first ask whether we can apply Kant's idea that laws of nature apply always in experience because they are preconditions of experience. This is now not an axiomatic statement but a working hypothesis. Which preconditions of experience would we accept a priori? If experience means to learn from the past for the future, then any empirical science presupposes an understanding of past and future. We consider past events as *facts*, future events as *possibilities*. We hypothetically try to do this by forming *two theses*, also to be called "*six words*":

A. Facts are discrete.
B. Possibilities are continuous.

This corresponds to the historical description of quantum theory given in Sect. 1.

But can the "six words" be considered as presuppositions of all possible experience? In Kant's view, preconditions of experience must be knowledge a priori. We cannot maintain that the two theses have this kind of a priori evidence; else classical continuum dynamics would never have been invented. Hydrodynamics, e.g., considers the continuously distributed field of velocities certainly as a continuous field of facts. Similarly, Schrödinger certainly considered his Ψ as a continuous factual field; this, precisely is the reason why the "collapse of Ψ" seems so paradoxical. If Ψ expresses only possibilities, then the collapse means the transition from possibility into fact. We have not yet tried to describe this in detail (see Sect. 4.4); but certainly it is a well-known everyday event and not a paradox. A new fact implies new possibilities. But all these considerations are not a priori in the sense of Kant. The continuous field of facts seems clearly acceptable to our intuition and our reasoning. So, what do we then mean by the two theses as preconditions of experience?

A first step towards an answer: In a circular epistemology, preconditions of experience do not need to be consciously present to our mind. In evolution, consciousness emerges from a sea of unconscious ways of behaviour. Modern psychology knows of the immense subconscious basis of our conscious perceptions. "Consciousness is an unconscious act". Thus the two theses may describe structures of organic or even inorganic nature which belong to the modes of time and thus are *objective* preconditions of experience without being known a priori.

Indeed: Possibilities can be quantified as probabilities, and these admit all real numbers between 0 and 1 as values; thus they can, and in an indeterministic theory even must be described in a continuum. If facts, on the other hand, are past events, we can only know them, when they have produced an irreversible process, leaving a document in nature or in our memory. Irreversible processes, however, lose their phase relation with neighbouring possible processes and thus become separated, i.e. discrete.

3 Quantum Logic and the Reconstruction of Quantum Theory

3.1 The Role of the Continuum

Traditionally, the structures of logic and of mathematics were considered as prior to and hence as independent of physics. Birkhoff and von Neumann were the first to realize that this must not necessarily be so in the case of logic. Hence we must now consider this step in the philosophy of quantum theory. But we shall see, that the step is connected with a change, perhaps not in the structure, but in the interpretation of the mathematical continuum. Hence we begin with a historical remark on this concept.

Aristotle defined "continuum" as a quantity which can be indefinitely subdivided into smaller quantities of the same structure. Geometry and motion, in modern terms space and time, were his examples; in this application, "continuum", in his philosophy, is a concept of physics. "Indefinitely" does not mean what since Cantor we would call "actually infinite"; it corresponds to the modern term "potentially infinite", to which mathematicians even in the time of Gauß and Cauchy adhered. Aristotle insisted (Physics, Book 8; cf. [19, Sect. IV.4.]) that, e.g., the path on which Achilles reaches the tortoise does not "consist of" infinitely many parts into which it is intellectually divided in the refutation of Zenos's Paradox. Physically, this path can be divided only by moving to the point where we want to divide it, coming to rest there, and then moving again; an activity which takes a finite time. Hence in a finite time-span we can only perform a finite number of divisions. This very consideration shows that Aristotle treats the continuum as a concept of physics.

When Cantor introduced the idea of actual infinity, and introduced the absolutely non-Aristotelian idea that a continuum is a non-countable set of points, he was criticized by philosophers and traditional mathematicians. They said: Number (called natural number in modern mathematics) expresses our ability of counting. Counting, as Kant, and in our century Brouwer pointed out, is done in time. We can count beyond any number reached in actual counting – in the language of the present paper a number reached by counting is a fact, the next numbers describe possibilities. Thus counting is indefinite but not actually infinite. Cantor cleverly replied: By your argument you admit that, while counted facts are always a finite set, the possible numbers form an infinite set, and mathematics is a science of concepts, i.e. of possibilities.

In quantum theory we describe possibilities as continuous. We shall have to investigate, whether Cantor's idea of the continuum as a point set then stays adequate.

3.2 Non-distributive Logic and Quantum Theory

The classical logic of propositions can be mathematically expressed in a Boolean algebra. This algebra is connected with set theory. For simplicity's sake we consider a finite set, e.g. of three elements a, b, c. They should express three mutually exclusive statements (in physics: time-dependent propositions like "the z-component of the angular momentum 1 is $+1$, or 0, or -1"; or they might, equivalently, mean the states whose presence is expressed in the corresponding statement). Any statements x,y,... permit the logical functions

$x \wedge y$: "x and y",
in set theory: the intersection of the subsets x and y,
$x \vee y$: "x or y",
in set theory: the join of the subsets x and y,
$\neg x$: "non x",
in set theory: the complement of the subset x.

The elements (in our example a, b, c) of the considered set are called the "atoms" of the lattice. For brevity we shall sometimes write xy for $x \vee y$. Then the corresponding Hasse diagram is given by Fig. 1.

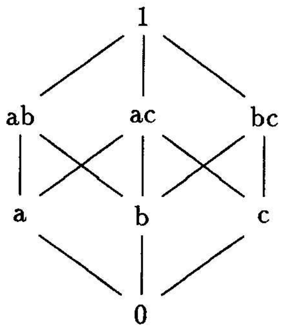

Fig. 1.. The lines in the diagram describe implications, e.g. $a < ab$, a is a subset of ab, or a implies ab. $1 = abc$ is the complete set, or "the true proposition", 0 is the null set, or "the false proposition".

In the logical interpretation referring to events in time, we shall also call the elements of a set containing n elements an "n-fold alternative": one of them must be true at any moment, and then none of the others can be true. We shall have to consider two theorems that hold in a Boolean lattice:

The first distributive law

$$xy \wedge z = (x \wedge z) \vee (z \wedge y), \tag{1}$$

and the law of double negation

$$\neg(\neg x) = x. \tag{2}$$

Instead of such a Boolean lattice, Birkhoff and von Neumann considered the lattice of the subspaces of a Hilbert space. In our example, let the Hilbert space be 3-dimensional, corresponding to the three values of spin-1 z-component. Here

$x \wedge y$ is again the intersection, now of the subspaces x and y,
$x \vee y$ is the subspace, linearly composed from x and y,
$\neg x$ is the subspace of all vectors orthogonal on x.

Now the atoms of the lattice are a continuously infinite set: all vectors of the given space. In this lattice, the law (2) still holds: every subset x is orthogonal on the subset consisting of all subsets which are orthogonal on x. But (1) does no longer hold in general. Let, e.g., x and y be two different vectors, and z a vector in their plane xy, but different from both x and y. Then:

$$xy \wedge z = z, \tag{3}$$

$$\text{but } x \wedge z = y \wedge z = (x \wedge z) \vee (y \wedge z) = 0\,. \tag{4}$$

The "logic" formulated in this kind of lattices is now generally called "quantum logic". We shall call it here, more specifically, "non-distributive logic".

There have been debates whether such a mathematical formalism deserves the name of "logic". We shall not enter into the philosophical backgrounds of this debate in the present paper. We only mention a few steps. Lorenzen has tried to deduce a constructive logic from rules of operational thinking [16] or of dialogue [17], thus excluding formal attempts like "quantum logic". Mittelstaedt [18] applied Lorenzen's methods to the description of physical experiments, thus justifying "quantum logic" as a logic. Weizsäcker [21, Chapt. 2] follows this path, describing it as a logic of temporal statements.

A different path was opened by Doebner and Lücke [5]. In the so-called "orthodox" tradition, as codified by von Neumann, the inevitability of probabilities in quantum mechanics has been interpreted as an inevitable indeterminism. Quantum logic was understood as a consequence and hence as corroboration of this indeterminism. Attempts at an experimental direct finding of deterministic hidden variables behind quantum mechanics have so far not been successful. But it seems still impossible, theoretically to exclude such a "hidden determinism" [2,1,22]. Our interpretation of CI as "minimal semantics" leaves this discussion open. Now Doebner and Lücke have shown that a deterministic hypothesis behind quantum mechanics can easily produce a non-distributive lattice, hence logic.

They have proved that a non-distributive logic can be directly embedded into a Boolean logic of "hidden variables". "Direct embedding" here means retaining the new relations $\wedge$ and $\vee$ from the full Boolean lattice. The method of "direct embedding", however, does not yet produce the full quantum theory. It does not produce the quantum-mechanical *violation of Bell's inequality.* Yet this can be achieved by adequate additional assumptions.

Doebner and Lücke have shown that their non-distributive quantum logic with direct embedding satisfies Bell's inequality, if we assume that the probabilities of combined, but independent measurements can be factorized into products of the probabilities of the separate measurements. In the simplest example this is easily shown. Consider two independent binary alternatives. Call the probability for the outcome a and b in the first alternative p_a, p_b, in the second alternative q_a, q_b, and the probability of an outcome x in the first *and* y in the second alternative r_{xy}. If the alternatives are independent (e.g., with a local interaction law, simultaneous and at a spatial distance), we would classically expect

$$r_{xy} = p_x q_y\,. \tag{11}$$

Now define "Bell's quantity" B_{xy} by

$$B_{xy} \overset{\text{def}}{=} p_x + q_y - r_{xx} + r_{yx} - r_{xy} - r_{yy} \overset{\text{def}}{=} p_x + q_y - C_{xy}\,. \tag{12}$$

From (11) we conclude

$$C_{xy} = +p_x q_x - p_y q_x + p_x q_y + p_y q_y = p_y(q_y - q_x) + p_x(q_y + q_x) = p_y(q_y - q_x) + p_x\,, \tag{13}$$

since

$$q_y + q_x = 1\,. \tag{14}$$

Further, due to (14)

$$q_y - q_x = 2q_y - 1\,, \tag{15}$$

and

$$B_{xy} = q_y + p_y - 2p_y q_y = p_y(1 - q_y) + q_y(1 - p_y) \geq 0\,. \tag{16}$$

This is Bell's inequality, valid since $1 \geq p_y \geq 0\,,\; 1 \geq q_y \geq 0\,.$

In this derivation, no use has been made of the assumption that the independence of the two results is produced by spatial distance; this is only one example of such independence. Since Bell's inequality seems now to be definitely violated in the special case of local distance, the preconditions of independence, where we introduce it, might equally lead to a wrong result. We tentatively conclude the *holism* of quantum theory: There are no strictly independent events.

We end this section by mentioning some unresolved problems. We have not investigated how far quantum logic implies full quantum theory. Non-distributive logic is certainly not sufficient. The basic additional point seems to be the symmetry which establishes pure states. Further, we have not found out how far a completed quantum theory admits a testable determinism in the assumed hidden variables. This would depend on an interpretation of these variables and their time-dependence. Accepting holism, they might just express the influence of the outer world on the object. This, again, would presuppose a theory of space and time in the quantum context.

In the following section we shall give a very brief outline of our own attempt at reconstructing quantum theory, including the theory of the space-time continuum, from simple postulates.

3.3 Reconstruction

The ensuing postulates try to formulate simple preconditions of quantum theory as a *theory of human knowledge*. This expresses the same tendency as our interpretation of CI as a *minimal semantics*. What, *at least*, is to be assumed for such a theory? (See [21, Chapt. 8] and [6,23]).

Postulates:

0. Holism.
   The reality is a whole (eine Ganzheit), not strictly separable into parts.
1. Alternatives.
   In a good approximation there are separable finite (n-fold) empirically decidable alternatives.
2. Indeterminism.
   Let x and y be two mutually exclusive states, then there are "intermediate" states z with conditional probabilities $p(z, x)$ and $p(z, y)$ of finding x or y if z is present, such that both probabilities are neither zero nor one.

3. Kinematics.
   States belonging to an alternative according to Post. 2 change continuously in time such that the conditional probabilities are not altered: $p[(x,t),(z,t)] = p[(x,0),(z,0)]$.

Consequences:

Assuming that any kinematical law fulfilling Post. 3 is permissible, we can conclude that Post. 2 implies a symmetry between the states z which permits the representation in a complex space with probabilities defined by a Hermitian metric. Thus "abstract quantum theory" is derived.

Any alternative then possesses a continuum of vectors, called "pure states" in traditional quantum theory. By the conditional probabilities $p(z,x)$ etc. these vectors are connected such that if z is present there is a probability to find x. In this sense the continuum of states expresses possibilities. In this direct experimental sense, z and x are not disjoint elements of a set but have a probability of being formal "identical". If "continuum" is considered as a concept of physics, expressing possibilities, it is an inadequate way of speech to call the continuum a set of "points". Of course, this expression is used by defining the "set of all states z" which belongs to a given discrete alternative. But then the question arises, which measurement would permit to distinguish two states z_1 and z_2, connected by $p(z_1,z_2) \neq 0,1$. It might be done by statistical measurements, with always a limited precision.

Our reconstruction does not exclude the possibility that behind the postulated indeterminism there is a deterministic lattice as studied by Doebner and Lücke. The present paper, as said before, cannot yet study the structures of the proposed hidden variables.

Another consequence is independent of this question. Every finite or countably infinite discrete alternative can be decided by successive decision of binary (2-fold) alternatives ("yes-no decisions"). The quantum theory of the binary alternative has the symmetry group $\mathrm{SU}(2) \times \mathrm{U}(1)$. $\mathrm{SU}(2)$ has a natural representation in a 3-dimensional real space, where $\mathrm{U}(1)$ may be used to describe time-dependence. We suppose that thus the three-dimensionality of position space is a necessary consequence of abstract quantum theory [21, Chapts. 9–10]. This, again, lies beyond the present paper; but it might be mentioned as indicating the probable universality of quantum theory.

4 Further Philosophical Questions

4.1 Minimal Semantics and Beyond

Describing CI as a minimal semantics of quantum theory, we try to keep free from "ideologies". Interpretations beyond CI are permissible if they can be formulated with sufficient clarity to admit a discussion, if possible even an experimental decision. It is, however, important to see which consequences can be drawn from the theory *without* adding new interpretations.

If the consideration on three-dimensional real space as a consequence of the possible reduction of all alternatives to successive binary alternatives should turn out to be successful, then the space-time continuum would not be "behind" quantum theory as a field in which hidden variables might be located, but would already be a consequence of the minimal semantics of the theory. Therefore we feel that this question ought to be intensively studied. It can easily be shown that the space-time continuum so deduced admits an approximate description by special relativity (in a tangential Minkowski space). Hence it will have as a consequence the existence of particles as irreducible representations of the Poincaré group. We would gladly invite able theoretical physicists to study these consequences. We suppose that they might give a frame in which the Doebner-Lücke background can be interpreted.

4.2 The Mind-Body Problem

If we interpret quantum theory correctly, the idea of two *substances*, viz. the thinking and the extended substance, as proposed by Descartes, is probably a misunderstanding. In a theory of human knowledge, there are two *roles* for parts of reality, the role of the knower and of the known, of subject and of object. The abstract quantum theory as reconstructed by postulates on decidable alternatives is essentially a theory on the time-dependence of *information*. Information can be defined as a measure for the quantity of *form*. Particles as representations of a symmetry group derived from abstract quantum theory are nothing but "agglomerations of form". But whose form is this? Plato considered form as the ultimate reality. Descartes' starting point was the self-awareness of the ego: I can doubt everything but not the fact that I am doubting. Modern psychology knows that this conscious psyche is embedded into an immensely larger sub- or unconscious psyche. Quantum theory does not imply but certainly also not exclude the idea that psyche or spirit is the basic reality. The "thinking substance", if it can know itself and thus decide alternatives on its own state, will, according to the considerations, necessarily also appear as "extended", at least to the approximation in which abstract quantum theory might be applied to these alternatives.

But why, then, are we finding ourselves as conscious beings isolated in an extended world where we so far see no other entities that might be regarded as possessing self-consciousness? Here it is to be remembered that our consciousness is a late step in evolution. It seems to presuppose a complicated organ, the brain and nervous system. The parts of the brain, the nervous cells, contribute to consciousness but seem not to have consciousness of their own. A nervous cell seems to have no private ego; neither has a special emotion, a sense of pain, a perception of colour, which is part of our self-awareness, an ego in its own. As we said before, consciousness rests on the subconscious psyche. Hence ego-consciousness, when it appears for the first time in evolution, must be seen to be unique, solitary in the extended world. What if we wait for another half billion of years of evolution? What if we were able to perceive the psychic aspects of larger parts of the universe?

4.3 Holism

In our postulates in Sect. 3.4, we started with a "zeroth" postulate of holism. As a consequence of quantum theory, this holism is fully recognized by many authors today. The state space of a composite object contains only a set of measure zero of product states in which its parts are in well-defined states of their own. Separate alternatives are thus no more than a useful manipulation of human conceptual thought. How would we have to describe their embedding in the real larger world of which we are only parts?

4.4 Events

We have so far left unconsidered the question how possibilities are transformed into facts. Haag has recently written a paper [12] in which he argues that a special postulate must be added to quantum theory which craves that this transition takes place. This problem was treated in [20] and [21, Chapt. 13.3]. We propose to treat this problem in a separate, later paper.

4.5 Final Remark

Philosophy is done today – not in the past, not in the future, not in eternity. It is reflection on what we seem to know *now*. Hence it may be justified to end with open questions.

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